

**Adding In-Plane Flexibility to the Equations of Motion
of a Single Rotor Helicopter**

Studies in Rotorcraft System Identification
NASA Ames Research Center Grant NAG2-561

Final Technical Report
February 1, 1989 to January 31, 2000

Principal Investigator
H.C.Curtiss, Jr.
Department of Mechanical and Aerospace Engineering
Princeton University
Princeton, NJ 08544

Technical Officer
Mark B. Tischler
NASA Ames Research Center
Flight Dynamics and Controls Branch, 211-5

February 6, 2000

INTRODUCTION

This report describes a way to add the effects of main rotor blade flexibility in the in-plane or lead-lag direction to a large set of non-linear equations of motion for a single rotor helicopter with rigid blades(1). Differences between the frequency of the regressing lag mode predicted by the equations of (1) and that measured in flight (2) for a UH-60 helicopter indicate that some element is missing from the analytical model of (1) which assumes rigid blades. A previous study (3) noted a similar discrepancy for the CH-53 helicopter. Using a relatively simple analytical model in (3), compared to (1), it was shown that a mechanical lag damper increases significantly the coupling between the rigid lag mode and the first flexible mode. This increased coupling due to a powerful lag damper produces an increase in the lowest lag frequency when viewed in a frame rotating with the blade. Flight test measurements normally indicate the frequency of this mode in a non-rotating or fixed frame. Note that an increase in the rotating-frame frequency corresponds to decrease in the lowest fixed-frame frequency (fixed-frame frequency = rotor speed-rotating-frame frequency). This fixed-frame frequency associated with lead-lag motion is often referred to as the regressing lag frequency. Another frequency appears in the fixed frame also associated with this rotating-frame frequency. It is the sum of the rotor speed and the rotating-frame frequency, and is called the advancing lag frequency. This frequency is usually above the bandwidth of the flight measurements and therefore is not considered further. The predicted change in frequency obtained by including in-plane flexibility brought the predicted value into good agreement with the measured value in (3). However, in this earlier study, this improvement was not incorporated into a complete set of equations of motion. It is the objective of this study to include blade flexibility in a full set of equations of motion and to compare the results with experiment for a different helicopter thus providing additional verification of effect noted in (3). A different explanation of this discrepancy is provided in (4), where it is suggested that the frequency difference is a result of a spring around the lag hinge. The spring would of course raise the rotating-frame lag frequency (thus reducing the fixed-frame frequency). Since there is no experimental evidence to suggest that a spring moment exists about the hinge on the CH-53, this explanation does not appear to be correct.

The importance of this modeling improvement is related to the fact that the regressing lag mode of the main rotor places limits on feedback gains that can be used in the design of flight control systems (5). It is important therefore that the characteristics of this mode be modeled accurately.

This report presents the additions necessary to the full equations of motion, to include main rotor blade lag flexibility. Since these additions are made to a very complex non-linear dynamic model, in order to provide physical insight, a discussion of the results obtained from a simplified set of equations of motion is included. The reduced model illustrates the physics involved in the coupling and should indicate trends in the full model. The simplified model suggests a simplification in the full model.

This final technical report describes work currently in progress. Appendix C lists the publications and reports that have been prepared under this grant.

DISCUSSION

Full Model

The full model here refers to the additions required to the equations of motion presented in (1) to include in-plane rotor blade flexibility. The resulting equations of motion are presented in Appendix I. The notation follows (1) and Figure A-1 shown the geometry involved. While, in general, the flap (out-of plane), lag (in-plane) and torsion motions of a twisted rotor blade are coupled, it is considered that a reasonable approach for the problem of interest is to assume that the in-plane flexibility can be considered without including the out-of-plane or torsional flexibility. In this initial approach, the numerical example is based on the in-plane stiffness at zero collective pitch. Certain details have been omitted such as the reduction in effective in-plane stiffness with increasing collective pitch. Also, only one flexible mode is employed. The results of (3) indicated one flexible mode should provide a satisfactory result if the mode is suitably chosen. The equations of motion are formulated so that it is a straight-forward process to include additional modes and their degrees of freedom. At some point in this investigation the effect of adding modes will be investigated as well as a more refined treatment of blade flexibility. Much insight can be gained concerning the importance of these refinements by using the reduced model described below.

The method of assumed modes (6) is used to develop the new equations of motion relating to the flexible motion of the blade. One flexible mode is introduced for each blade. A single coordinate, the tip deflection, describes the elastic deflection that arises due to flexibility (Fig A-1). The deflection along the blade is described by a mode shape which is assumed to be known. Selection of this mode is discussed below. These new coordinates (one for each blade) result in an additional number of new equations equal to the number of blades. In addition, there will be new terms added the existing "rigid" lag equation for each blade (pg. 5.1-31 of (1)). Note that these equations referred to are all written in a rotating frame (i.e., moving with the blade). At some point in the analysis the

motion variables are converted to a non-rotating (or fixed) frame by the multi-blade coordinate relationships given on pg. 5.1-19 of (1). Similar relationships can be used for the new coordinates corresponding to the flexible deflection. See Appendix I for the definitions of quantities from (1) related to the problem of interest here. The primary effect of these additional degrees of freedom (associated with blade flexibility) on the helicopter motion is expected to be through their contribution the in-plane inertial shears. Typically, the aerodynamic forces associated with lag motion are relatively small and may be neglected. In adding terms and degrees of freedom to a very complex set of equations it seems highly desirable to proceed in small steps, by adding a few terms, and then interpreting the changes before proceeding to add more. At some point, all the aerodynamic effects should be added to the equations. Appendix I provides the details. The aerodynamic forces on the blade would primarily be affected by the additional in-plane velocity due to flexibility.

The flexible mode shape employed in the example is selected to be a cantilever mode. This is shown as a good choice for the problem of interest in (3). Physically it may be noted that in the limit of an infinitely strong lag damper, the blade will be a cantilever beam. The rigid motion can be considered as an articulated mode. When the lag damping is small, the cantilever mode will combine with the rigid mode to produce two modes: one is approximately the rigid mode as the coupling is weak when the damping is small; the other is an approximation to the first articulated mode.

It may be noted that the flexible blade equation is not used in the form it appears after the derivation. It is combined with the rigid blade equation so that a simplification is possible. To summarize, the full model involves :

1. Two terms added to the rigid lag equation (A-1)
2. One new equation for each blade (A-3)
3. Terms added to the inertial shears (A-5) and (A-6)
4. One term added to the blade velocity (A-4)
5. Correction of an error in (1), in the inertial shear (A-7)

It is suggested that the initial approach consist of adding items 1,2, and 3 (in a simplified form (A-5S, A-6S)). Once these additions are producing satisfactory results, the other additions can be made. Also, investigations with the reduced model suggest that the term in (A-3) proportional to the flexible lag acceleration can be neglected. This simplification is called the "quasi-static model" here. Note, that the flexible lag acceleration term in (A-1) should not be dropped.

Reduced Model

In order to obtain some understanding of the physics of this problem, and to have an indication of the trends to be expected with the full model, a reduced or simplified model is presented in Appendix B. This reduced model is used to investigate the variations in the frequency and damping of the modes due to changes in blade in-plane stiffness and mechanical lag damping. This reduced model is obtained from the full model based on the following assumptions:

1. Aerodynamic forces are neglected. Only the effect of cyclic pitch input is retained as this is a significant source of excitation of the lag motion. Generally, the effect of aerodynamic forces are small in the lag motion. This is, of course, one of the reasons for the mechanical lag damper as the aerodynamic damping tends to be small.
2. No body motion. The coupling between lag motion and body motion tends to be weak.
3. Flap motion may be neglected. This assumption is satisfactory for our purposes here, but should be used with care. There is significant coupling between lag and flap although generally the frequencies are not significantly affected.
4. Constant rotor speed.
5. Lag and flap angles are small so that the equations of motion can be linearized.

These assumptions lead in the rotating frame to a set of two linear second-order differential equations describing the coupled dynamics of the rigid lag motion and the flexible lag deflection (equations (B-1) and (B-3)). A fourth order system is obtained as shown in Appendix B (B-1", B-3"). Numerical results are presented using the mass and stiffness distributions listed in Table B-1 which approximate the characteristics of the UH-60 main rotor blades. Mode shapes correspond to rigid lag motion and cantilever bending. When the lag damping (D) is small (less than about 5 for this example), there is only weak coupling. The rigid mode is little changed by the addition of the flexible mode and there are two lightly damped oscillatory modes: one near the rigid mode (7 rad/sec); the other is the articulated mode (154 rad/sec). As the strength of the lag damping is increased, the modes are altered. The low frequency changes are shown in Figure B-1a, a root locus for lag damping (D) increasing from zero to 25. The increase in lag damping initially causes a frequency reduction as would be expected (to approximately $D=11$, the value indicated by Forecast results). but as the lag damping is increased further, the frequency becomes larger than that of the rigid mode, as the locus tends toward the cantilever mode (at 22.6 rad/sec). Figure B-1b compares the one-degree-of-freedom rigid-blade model (B-3R) with the two-degree-of-freedom model,

showing the modal frequency and damping differences as the mechanical damping increases. Compare the large difference in the frequency and modal damping at a lag damping of $D=12$ (Figures B-1a and B-1b). If the stiffness of the blade ($H=F2$) is reduced, thus lowering the cantilever frequency, this difference, a result of the coupling between the rigid and flexible modes will become more pronounced at lower values of D . Figure B-2 shows a root locus for variation of the blade stiffness parameter ($H=F2$) which is proportional to the in-plane stiffness of the blade. With $F2$ at its estimated value (445), the lowest mode is very close to the rigid mode. As $F2$ is reduced, reflecting a decrease in the in-plane stiffness it can be seen that after a small range where the frequency decreases, the frequency of this mode increases, and the damping decreases. For values of $F2$ below 300, there is a significant frequency increase, and the modal damping is very small at $F2=50$. This trend should be evident in the full model calculations.

The reduced equations can be further simplified by noting that the coefficient of the flexible lag acceleration in the flexible lag equation is small. Neglecting this term makes little change in the trends shown in Figures B-1 and B-2. The sketches in Figure B-3 illustrate the branch of the locus that is neglected. The flexible acceleration term in the rigid equation however cannot be neglected without significantly changing the trends shown in Figures B-1 and B-2.

The quasi-static equations of motion are also presented in a fixed or non-rotating frame on page B-6 (B-3F).

SUMMARY

A method is presented to add in-plane flexibility to the equations of motion for a single rotor helicopter with rigid blades. A reduced set of equations of motion is included to assist in interpreting the effects in the full equations of motion due to this addition. The sensitivity of the results to certain simplifications made in the approach, not considered to be of primary importance, should be investigated. Specifically, limiting the approach to one flexible mode should be examined. This can be done with the reduced model. A refined calculation of numerical parameters such as the in-plane flexibility, to include the effect of collective pitch and twist is desirable.

REFERENCES

1. Howlett, J.J., UH-60A Black Hawk Engineering Simulation Program :Volume I- Mathematical Model, NASA Contractor Report 166309, December 1981
2. Fletcher, Jay W., A Model Structure for Identification of Linear Models of the UH-60 Helicopter in Hover and Forward Flight, NASA TM 110362, August 1995

3. Hong,S.W.and Curtiss,H.C., An Analytic Modeling and System Identification Study of Rotor/Fuselage Dynamics in Hover, AHS Specialists' Conference on Piloting Vertical Flight Aircraft, San Francisco,CA, January 1993
4. Aponso,B.L.,et al, Identification of Higher-Order Helicopter Dynamics Using Linear Modeling Methods,47th Annual Forum of the AHS, Phoenix, Az, May 1991
5. Curtiss,H.C., Stability and Control Modeling, paper no.41,12th European Rotorcraft Forum,Garmisch-Partenkirchen, Germany, September 1986
6. Meirovitch,Leonard, Elements of Vibration Analysis, Second Edition, McGraw-Hill Book Co.,1986,pp282-285
7. Curtiss,H.C., Physical Aspects of Rotor Body Coupling in Stability and Control,46th Annual Forum of the AHS, Washington,DC, May 1990

APPENDIX A

FULL MODEL

EQUATION FOR $\ddot{\delta}_{IB}$ HAS TWO NEW TERMS (pg 5.1-31)

$$\ddot{\delta}_{IB} = \frac{M_b}{I_b \cos \beta_{IB}} [F] + [G] + \left[\frac{M_{LLO} + M_{LFO}}{I_b \cos^2 \beta} \right]_{IB} - \left[\frac{M_{LAB}}{I_b \cos \beta} \right]_{IB} \\ - \frac{M_{12}}{I_b \cos \beta_{IB}} \ddot{\delta}_{FIB} - \frac{K_{12}}{I_b \cos \beta_{IB}} \delta_{FIB} \quad (A-1)$$

UNDERLINED TERMS ARE NEW.

NEW EQUATION FOR $\ddot{\delta}_{FIB}$ FOR EACH BLADE

AS DERIVED:

$$\ddot{\delta}_{FIB} = - \frac{K_{22}}{M_{22}} \delta_{FIB} + \frac{M_{b2}}{M_{22}} \left[F + \sin \delta_{IB} [e(r_s - \Omega^2)] \right] \\ + \frac{M_{12}}{M_{22}} [G] - \left[\frac{M_{LAB2}}{M_{22}} \right]_{IB} - \frac{M_{12}}{M_{22}} \ddot{\delta}_{IB} - \frac{K_{12}}{M_{22}} \delta_{IB} \quad (A-2)$$

IT IS CONVENIENT TO USE ANOTHER FORM FOR THIS EQUATION, RELATED BY:

$$(A-3) = \frac{M_{22}}{M_{12}} (A-2) - (A-1)$$

$$\left[\frac{M_{22}}{M_{12}} - \frac{M_{12}}{I_b \cos \beta_{IB}} \right] \ddot{\delta}_{FIB} = - \left[\frac{K_{22}}{M_{12}} - \frac{K_{12}}{I_b \cos \beta_{IB}} \right] \delta_{FIB} \quad (A-3) \\ + \left[\frac{M_{b2}}{M_{12}} - \frac{M_b}{I_b \cos \beta_{IB}} \right] F + \frac{M_{b2}}{M_{12}} (\sin \delta_{IB} (e(r_s - \Omega^2))) \\ - \left[\frac{M_{LLO} + M_{LFO}}{I_b \cos^2 \beta} \right]_{IB} - \left[\frac{M_{LAB2}}{M_{12}} - \frac{M_{LAB}}{I_b \cos \beta} \right]_{IB} - \frac{K_{12}}{M_{12}} \delta_{IB}$$

(A-1)

ADD THE INFLUENCE OF FLEXIBLE BLADE VELOCITY TO U_{TI} (pg S.1-22):

$$\Delta U_{TI} = \left\{ \frac{\Phi_2 |_{\text{CENTER OF SEGMENT}}}{\Omega_T} \right\} \dot{\delta}_{FIB} \cos \beta_{IB} \quad (A-4)$$

ADD TO INERTIAL SHEARS: (pg S.1-33)

$$\Delta F_{YI} = M_{b2} \left[\cos \delta \left(2 \dot{\delta}_F (\Omega - r_s) + \delta_F \ddot{\delta} + 2 \dot{\delta}_F \dot{\delta} \right) + \sin \delta \left(\ddot{\delta}_F - (\Omega - r_s)^2 \delta_F - 2(\Omega - r_s) \dot{\delta}_F \dot{\delta} - \delta_F \dot{\delta}^2 \right) \right]_{IB} \quad (A-5)$$

$$\Delta F_{XI} = M_{b2} \left[\cos \delta \left(-\ddot{\delta}_F + (\Omega - r_s)^2 \delta_F + 2(\Omega - r_s) \dot{\delta}_F \dot{\delta} + \delta_F \dot{\delta}^2 \right) + \sin \delta \left(2 \dot{\delta}_F (\Omega - r_s) + 2 \dot{\delta}_F \dot{\delta} + \delta_F \ddot{\delta} \right) \right]_{IB} \quad (A-6)$$

IN ADDITION THERE IS AN ERROR IN THE INERTIAL SHEAR EXPRESSION IN CR 66309. THE FOLLOWING TERMS ARE MISSING: pg S.1-33

$$\Delta F_{Y_{IB}} = M_b \left[-2 \sin \beta_{IB} \sin \delta_{IB} \left(\dot{\delta}_{IB} \dot{\beta}_{IB} - \dot{\beta}_{IB} (r_s - \Omega) \right) + \cos \beta_{IB} \sin \delta_{IB} \left(\dot{\Omega} - \dot{r}_s \right) \right] \quad (A-7)$$

THUS THE FULL MODEL INVOLVES:

- 1.) ADDING 2 TERMS TO THE EXISTING RIGID BLADE EQUATIONS, EQN (A-1)
- 2.) ADDING NEW EQUATIONS OF MOTION FOR δ_{FIB} , EQN (A-3)
- 3.) ADDING TERM TO BLADE VELOCITY, EQN (A-4)
- 4.) ADDING TERMS TO INERTIAL SHEARS, EQNS (A-5) AND (A-6)
- 5.) CORRECT INERTIAL SHEAR EQUATION, EQN (A-7)

SINCE THIS PROBLEM IS VERY COMPLEX, A STEP-BY-STEP APPROACH IS DESIRABLE.

FIRST

(a) ADD TERMS TO EQU (A-1)

(b) ADD EQUATIONS (A-3)

(c) ADD SIMPLIFIED FORM OF EQUATIONS (A-5), (A-6)

$$\Delta F_{YI} \cong M_{b2} (2\dot{\delta}_F (\Omega - r_s)) \quad (A-5S)$$

$$\Delta F_{XI} \cong M_{b2} (-\ddot{\delta}_F + (\Omega - r_s)^2 \delta_F) \quad (A-6S)$$

SECOND

d.) ADD FULL INERTIAL SHEAR TERMS REPLACE (A-5S) AND (A-6S) BY (A-5), (A-6)

THIRD

e.) CORRECT INERTIAL SHEAR (A-7)

f.) ADD TERM TO BLADE VELOCITY (A-4)

STEPS d.), e.) AND f.) SHOULD RESULT IN ONLY SMALL CHANGES.

NOTATION

$$F = \sin \delta_{IB} \left\{ \dot{V}_{YS} \sin \psi_{IB} - \dot{V}_{XS} \cos \psi_{IB} - e(r_s - \Omega)^2 \right\} \\ - \cos \delta_{IB} \left\{ \dot{V}_{XS} \sin \psi_{IB} + \dot{V}_{YS} \cos \psi_{IB} + e(\Omega - \dot{r}_s) \right\} \quad \text{SEE S.1-31}$$

$$G = \frac{\sin \beta_{IB}}{\cos \beta_{IB}} \left[2\dot{\beta}_{IB} (\Omega + \dot{\delta}_{IB} - r_s) + \dot{q}_s \sin(\psi + \delta)_{IB} \right. \\ \left. - \dot{p}_s \cos(\psi + \delta)_{IB} \right] + (\dot{r}_c - \dot{\Omega}) \quad \text{SEE S.1-31} \\ + 2\dot{\beta}_{IB} \left[\cos \delta_{IB} (q_s \sin \psi_{IB} - p_s \cos \psi_{IB}) + \right. \\ \left. \sin \delta_{IB} (p_s \sin \psi_{IB} + q_s \cos \psi_{IB}) \right]$$

M_{LAB} AERODYNAMIC TERMS (pg S.1-28)



$$M_{LAB2} \equiv R_T \sum_{TS=1}^{NSS} \Phi_2 \left| F_{TI} \right|_{\text{CENTER OF SEGMENT}}$$

NEW TERM IN (A-3)

$$\frac{Z}{R} = \delta_{IB} \Phi_1 + \delta_{FIB} \bar{\Phi}_2 \quad \text{BLADE LAGWISE DEFLECTION}$$

δ_{IB} LAG ANGLE AS DEFINED (pg 5.1-8)

δ_{FIB} FLEXIBLE DEFLECTION (SEE FIG A-1)

Φ_1 = RIGID MODE SHAPE $0 \rightarrow \xi, \phi_1 = 0; \xi \rightarrow 1, \phi_1 = (X - \xi) \quad 0 < X < 1$

$\bar{\Phi}_2$ = CANTILEVER MODE SHAPE $0 - \xi, \phi_2 = 0; \xi - 1,$

$$\xi = \text{HINGE OFFSET} / R_T \quad \phi_2 = 2 \left(\frac{X - \xi}{1 - \xi} \right)^2 - \frac{4}{3} \left(\frac{X - \xi}{1 - \xi} \right)^3 + \frac{1}{3} \left(\frac{X - \xi}{1 - \xi} \right)^4$$

M_{LLD} MECHANICAL LAG DAMPER. GEOMETRY DEFINED ON pg 5.1-10. CHARACTERISTICS GIVEN ON 5.1-55/56. NOTE THAT VERTICAL SCALE IS WRONG. IT SHOULD READ $FLDMR \times 10^{-1}$, i.e., MAXIMUM READING ON 5.1-55 IS ABOUT 4000 LBS. TABLE ON 5.1-64 OK.

$$\bar{m} = m R_T \quad \text{BLADE RUNNING MASS} \times R_T$$

$$M_{ij} = R_T^2 \int_0^1 \bar{m} \phi_i \phi_j dx \quad x = \frac{r}{R}$$

$$K_{ij} = \underbrace{\int_0^1 \frac{EI}{R_T} \phi_i'' \phi_j'' dx}_{\text{ELASTIC TERM}} + R_T^2 (\Omega - \tau_s)^2 \left(\underbrace{\int_{\xi}^1 \bar{m} x \int_{\xi}^x \phi_i' \phi_j' d\xi dx - \int_{\xi}^1 \bar{m} \phi_i \phi_j dx}_{\text{CENTRIFUGAL STIFFENING}} \right)$$

NOTE THAT FOR MODE SHAPES SELECTED:

$$M_{11} = I_b$$

FOR APPROXIMATE MASS DISTRIBUTION THIS IS NOT QUITE TRUE FOR NUMERICAL EXAMPLE M_{11} USED IN CALCULATION OF N , I_b OTHERWISE IS USED

(A-4)



BLADE ELASTICITY APPEARS ONLY IN K_{22}

$$K_{11} = (\Omega - \gamma_s)^2 \int M_b R_T \quad M_b \text{ IS BLADE 1ST MASS MOMENT}$$

NOTE THAT

$$K_{11}, K_{12} = K_{21} \text{ CENTRIFUGAL STIFFENING ONLY.}$$

$$M_{b2} = R_T \int_0^1 \bar{m} \phi_2 dx$$

COMPARE:

REFERENCE (1)

$$M_{11} = 1612.6 \text{ SWG-FT}^2$$

$$I_b = 1512.6 \text{ SWG-FT}^2$$

$$K_{11} = 103.8 \Omega^2 \text{ FT-LBS}$$

$$e M_b \Omega^2 = 108.4 \Omega^2 \text{ FT-LBS}$$

(THESE SHOULD BE THE SAME; MASS DISTRIBUTION APPROXIMATION)
YIELDS SMALL DIFFERENCES

RESULTING TERMS ARE:

$$M_{11} = 1612.6 \text{ SWG-FT}^2$$

$$M_{22} = 1276.4 \text{ SWG-FT}^2$$

$$M_{12} = M_{21} = 1419.5 \text{ SWG-FT}^2$$

$$\Omega = 27 \text{ RAD/SEC.}$$

$$K_{11} = 103.8 \Omega^2 \text{ FT-LBS}$$

CF STIFFENING ONLY

$$K_{22} = 4.656 \times 10^5 + 314.3 \Omega^2 \text{ FT-LBS} \cdot \text{ELASTIC} + \text{CF}$$

$$K_{21} = K_{12} = 92.87 \Omega^2 \text{ FT-LBS} \text{ CF STIFFENING ONLY}$$

THESE ARE USED IN APPENDIX B. TABLE B-1 LISTS MASS AND STIFFNESS DISTRIBUTIONS USED.

CHECK NUMBERS WITH UNIFORM BEAM RESULTS.

ARTICULATED MORE, NON-ROTATING

$$A = \frac{EI_0}{m_0 R_T^4} = 53.3$$

$$\sqrt{\frac{H_0}{N A}} = 16.7 \left[\begin{array}{c} \text{UNIFORM CANTILEVER} \\ 15.3 \end{array} \right]$$

$$\sqrt{\frac{K_{22}}{M_{220} A}} = 2.62 \left[\begin{array}{c} \text{UNIFORM CANTILEVER 2.52} \\ \text{ELASTIC ONLY} \end{array} \right]$$

$$\frac{314.3}{1276.4} = .246 \left[\begin{array}{c} \text{SOUTHWELL COEFF} \\ \text{CHECKS TN 3453} \end{array} \right]$$

APPENDIX B

REDUCED MODEL

ASSUMPTIONS

a.) AERODYNAMIC FORCES NEGLECTED

$$M_{LAB} = 0, M_{LAB2} = 0$$

NORMALLY THESE PRODUCE A SMALL LAG DAMPING

b.) NO BODY MOTION

$$\dot{V}_{XS} = \dot{V}_{YS} = \dot{r}_S = r_S = \dot{q}_S = \dot{p}_S = \dot{r}_G = p_S = q_S = 0$$

c.) NEGLECT FLAP MOTION, FLAP ANGLE SMALL

$$\dot{\beta}_S = 0, \cos \beta_S \approx 1, \sin \beta_S \approx \beta_S$$

d.) CONSTANT ROTOR SPEED

$$\dot{\Omega} = 0$$

e.) LAG ANGLE SMALL

$$\sin \delta_{IB} \approx \delta_{IB}, \cos \delta_{IB} \approx 1$$

CONSEQUENTLY

$$F = -e\Omega^2 \delta_{IB}$$

$$G = 0$$

ALSO NOTE $M_{FLD} = 0$

LINEARIZE M_{LID}

DAMPER KINEMATICS PG. S.1-30 LINEARIZED

$$\frac{\partial x}{\partial \delta_{IB}} = \frac{\partial \dot{x}}{\partial \dot{\delta}_{IB}} = 9.16''$$

DAMPER MAP PG S.1-56

SLOPE THRU ORIGIN $\frac{\partial F_D}{\partial \dot{x}} = 1125 \frac{\text{LBS}}{\text{IN/SEC}}$

$$\frac{\partial M_{LID}}{\partial \dot{\delta}_{IB}} \approx -(9.15'') \frac{\partial F_D}{\partial \dot{x}} \frac{\partial \dot{x}}{\partial \dot{\delta}_{IB}} \frac{1}{12} = -7858 \frac{\text{FT-LBS}}{\text{RAD/SEC.}}$$

↑
MOMENT ARM, IN. FROM S.1-30

$$\text{DENOTE } D = -\frac{1}{I_b} \frac{\partial M_{LLD}}{\partial \dot{\delta}_{IB}}; M_{LLD} = \frac{\partial M_{LLD}}{\partial \dot{\delta}_{IB}} \dot{\delta}_{IB} \quad \left\{ D = 5.2 \text{ FROM THIS CALCULATION} \right\}$$

WITH THESE ASSUMPTIONS:

EQUATION (A-1) BECOMES

$$\ddot{\delta}_{IB} = -\frac{eM_s\Omega^2}{I_b} \delta_{IB} - D \dot{\delta}_{IB} - \frac{M_{12}}{I_b} \ddot{\delta}_{FIB} - \frac{K_{12}}{I_b} \delta_{FIB} \quad (B-1)$$

(B-1) IS REDUCED VERSION OF (A-1)

EQUATION (A-2) BECOMES

$$\ddot{\delta}_{FIB} = -\frac{K_{22}}{M_{22}} \delta_{FIB} - \frac{M_{12}}{M_{22}} \ddot{\delta}_{IB} - \frac{K_{12}}{M_{22}} \delta_{IB} \quad (B-2)$$

EQUATION (A-3) BECOMES

$$\left(\frac{M_{22}}{M_{12}} - \frac{M_{12}}{I_b} \right) \ddot{\delta}_{FIB} = - \left[\frac{K_{22}}{M_{12}} - \frac{K_{12}}{I_b} \right] \delta_{FIB} + \left[\frac{K_{11}}{I_b} - \frac{K_{12}}{M_{12}} \right] \delta_{IB} + D \dot{\delta}_{IB} \quad (B-3)$$

$\uparrow \left(\text{OR } \frac{eM_s\Omega^2}{I_b} \right)$

INTRODUCE A MORE COMPACT NOTATION:-

$$N = \frac{M_{22}}{M_{12}} - \frac{M_{12}}{M_{11}}$$

$$H = \frac{K_{22}}{M_{12}} - \frac{K_{12}}{I_b}$$

$$J = \frac{K_{11}}{I_b} - \frac{K_{12}}{M_{12}} \cong \frac{eM_s\Omega^2}{I_b} - \frac{K_{12}}{M_{12}}$$

$$P = \frac{M_{12}}{I_b}$$

(B-2)

$$Q = \frac{K_{12}}{I_b}$$

NOTE THAT $J + \frac{Q}{P} = \omega_R^2$ RIGID BLADE UNCOUPLED FREQUENCY²

$$\left(\frac{H + Q}{N + P} \right) = \omega_c^2 \text{ CANTILEVER FREQUENCY}^2$$

COMPARE GREG S. NOTATION 1-27-2000 E-MAIL

$H = F2$ BLADE STIFFNESS IS MAJOR CONTRIBUTOR

$\frac{Q}{P} = F8$ $N=0$ QUASI-STATIC MODEL

D IS LINEARIZED $F6$

$F = F4 = -e\Omega^2 \delta_{IB}$ IN REDUCED CASE

$J = F3 * F4 + F5 - F8$

QUASI STATIC EON

$$DFLX = (F3 * F4 + F5 - F6 - F7 - F8 * DW4) / F2$$

$F7 = \text{AERODYNAMICS}$ SET=0 REDUCED MODEL
"QUASI-STATIC"

$$\delta_F = \left(+ \frac{eM_b\Omega^2}{I_b} \delta_{IB} + D \dot{\delta}_{IB} - \frac{Q}{P} \delta_{IB} \right) / H$$

$$\delta_F = \left(\left[\frac{eM_b\Omega^2}{I_b} \right] - \frac{Q}{P} \right) \delta_{IB} + D \dot{\delta}_{IB} / H \quad (\text{GREG S. Q.S.})$$

INTRODUCING NEW NOTATION

$$\ddot{\delta}_{IB} = \left(J + \frac{Q}{P} \right) \delta_{IB} - D \dot{\delta}_{IB} - P \ddot{\delta}_{FIB} - Q \delta_{FIB} \quad (B-1')$$

$$N \ddot{\delta}_{FIB} = -H \delta_{FIB} + J \delta_{IB} + D \dot{\delta}_{IB} \quad (B-3')$$

((B-3))

NOTING THAT $J + \frac{Q}{P} = \omega_R^2$ AND REORGANIZING

$$\ddot{\delta}_{IB} + D \dot{\delta}_{IB} + \omega_R^2 \delta_{IB} + P \ddot{\delta}_{FIB} + Q \delta_{FIB} = 0 \quad (B-1'')$$

$$-D \dot{\delta}_{IB} - J \delta_{IB} + N \ddot{\delta}_{FIB} + H \delta_{FIB} = 0 \quad (B-3'')$$

WITH $N=0$ (B-3'') REDUCES TO QUASI-STATIC EQN OF GREG'S.

WE SEE THAT THE LINEARIZED EQNS ARE 2 COUPLED SECOND ORDER EQUATIONS. (B-1'') AND (B-3'')

USING VALUES OF K_{ij} , M_{ij} AS GIVEN BY HCC NOTES 11-5-99 FOR VARIABLE MASS / STIFFNESS ALSO IN GREG'S E-MAIL 11-17-99

$$N = .0189 \quad P = .939$$

$$EI = 444.7 \frac{1}{\text{SEC}^2} \quad Q = 44.75 \frac{1}{\text{SEC}^2}$$

$$J = 2.34 \frac{1}{\text{SEC}^2} \quad D = 10.8 \quad \text{THIS VALUE WAS TAKEN TO MATCH FORECAST. IT IS ABOUT TWICE THE LINEARIZED VALUE DEVELOPED ABOVE EQN PAGES B-1, B-2}$$

$$\omega_R^2 = 50.03 \left(\frac{\text{RAD}}{\text{SEC}} \right)^2$$

TAKING THE LAPLACE TRANSFORM

$$\begin{bmatrix} (s^2 + Ds + \omega_R^2) & (Ps^2 + Q) \\ (-Ds - J) & (Ns^2 + H) \end{bmatrix} \begin{Bmatrix} \delta_{IB} \\ \delta_{FIB} \end{Bmatrix} = 0 \quad (B-3T)$$

$$1: (s^2 + Ds + \omega_R^2) = 0 \quad \text{RIGID LAG CHARACTERISTIC EQUATION}$$

$$\text{ROOTS } -5.4 \pm 4.57i \quad ; D=0 \quad \pm 7.07i \quad (B-3R)$$

$$(Ns^2 + H) = 0 \quad \text{APPROX 1ST FLEXIBLE MODE OF ARTICULATED BLADE}$$

$$\text{ROOTS } \pm 153i \quad \text{HIGH FREQUENCY}$$

ROOTS OF FULL SYSTEM NO DAMPING

$$(s^2 + \omega_R^2)(Ns^2 + H) + J(Ps^2 + Q) \quad \pm 154i \quad ; \pm 7.07i$$

CLOSE PROXIMITY TO UNCOUPLED MODES SUGGESTS WEAK COUPLING.

WITH $D=10.8$ ROOTS ARE $-500.9, -30.7$

DAMPING HAS STRONG EFFECT $-7.8 \pm 4.07i$
ON H_1 -FREQUENCY MODE
AND ALSO CHANGES RIGID MODE

ARRANGE FOR ROOT LOCUS STUDIES, VARY D, H

$$\begin{aligned} (s^2 + Ds + \omega_R^2)(Ns^2 + H) + (Ds + J)(Ps^2 + Q) \\ + \Delta D s [(N + P)s^2 + H + Q] \\ + \Delta H [s^2 + Ds + \omega_R^2] = 0 \end{aligned} \quad (B-4)$$

$\Delta H \rightarrow \infty$ RIGID BLADE APPROACHES RIGID MODE

$\Delta D \rightarrow \infty$ ROOT BECOMES FIXED SO CANTILEVER

FREQUENCY IS $\sqrt{\frac{H+Q}{N+P}} = 22.6 \text{ rad/sec.}$

$N \approx 0$ GIVES QUASI-STATIC RESULTS THAT ELIMINATE HIGH
THE HIGH FREQUENCY ($H \approx 4i$). THE SYSTEM IS REDUCED
FROM FOURTH ORDER TO THIRD ORDER.

$N=0$ QUASI-STATIC CHARACTERISTIC EQN.

$$\left(s + \frac{J}{D}\right) \left(s^2 + \frac{Q}{P}\right) + \frac{H}{DP} (s^2 + Ds + \omega_R^2) = 0 \quad (B-5)$$

FOR $H=0$ ROOTS $-\frac{J}{D}; \sqrt{\frac{Q}{P}}i$

$H = \infty$ RIGID BLADE

USING EQUATION (B-5) IN PLACE OF (B-4) DOES NOT
SIGNIFICANTLY CHANGE THE MODES OF INTEREST.

(B-5)

EQUATION SUMMARY (QUASI-STATIC)

ROTATING FRAME

$$\begin{bmatrix} (s^2 + Ds + \omega_k^2) & (Ps^2 + Q) \\ (-Ds - J) & H \end{bmatrix} \begin{Bmatrix} \delta_{IB} \\ \delta_{FIB} \end{Bmatrix} = 0 \quad (B-3QS)$$

THIRD ORDER SYSTEM

ONE REAL ROOT + OSCILLATORY ROOTS
 λ_F , $\sigma_F \pm i\omega_F$

FIXED FRAME (QUASI-STATIC)

$$\begin{bmatrix} s^2 + Ds + (\omega_k^2 - \Omega^2) & (2s + D)\Omega & Ps^2 + (Q - P\Omega^2) & -2P\Omega s \\ -(2s + D)\Omega & (s^2 + Ds + (\omega_k^2 - \Omega^2)) & 2P\Omega s & Ps^2 + (Q - P\Omega^2) \\ -(Ds + J) & -D\Omega & H & \\ D\Omega & -(Ds + J) & & H \end{bmatrix} \begin{Bmatrix} A_{IL} \\ B_{IL} \\ A_{IF} \\ B_{IF} \end{Bmatrix} = 0 \quad (B-3F)$$

SEE PG 5.1-19(1)

$$A_{IL} = \frac{2}{b_s} \sum \delta_{IB} \cos \psi_{IB}$$

$$A_{IF} = \frac{2}{b_s} \sum \delta_{FIB} \cos \psi_{IB}$$

$$B_{IL} = \frac{2}{b_s} \sum \delta_{IB} \sin \psi_{IB}$$

$$B_{IF} = \frac{2}{b_s} \sum \delta_{FIB} \sin \psi_{IB}$$

SIXTH ORDER SYSTEM WITH CHARACTERISTIC ROOTS

$$(\sigma_R \pm i\omega_R)_1 = \lambda_F \pm i\Omega$$

$$(\sigma_R \pm i\omega_R)_2 = \sigma_F \pm (\omega_F + \Omega)i \quad \text{"ADVANCING MODE"}$$

$$(\sigma_R \pm i\omega_R)_3 = \sigma_F \pm (\Omega - \omega_F)i \quad \text{"REGRESSING MODE"}$$

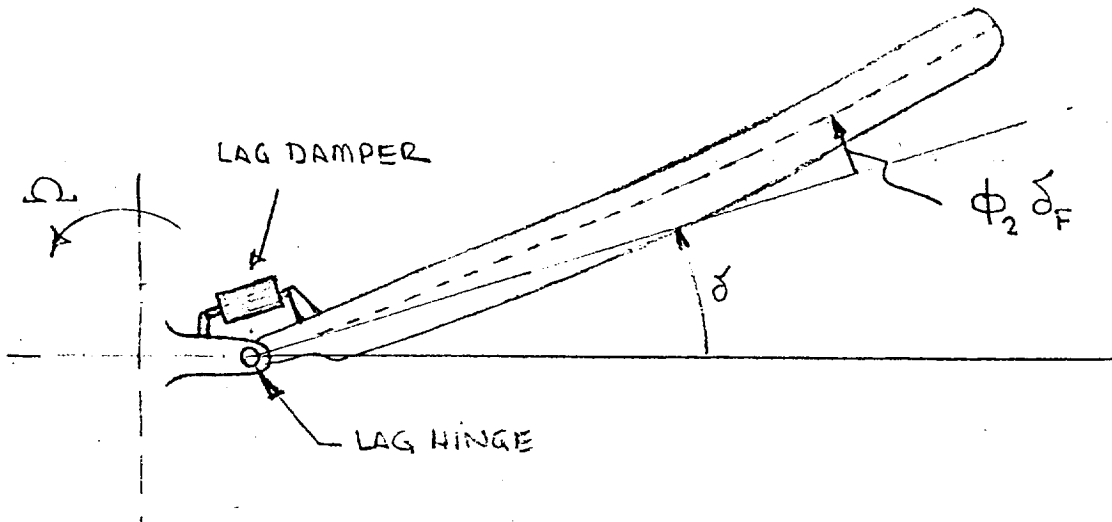


FIGURE A-1 RIGID AND ELASTIC DEFLECTION
GEOMETRY OF PROBLEM

CHORDWISE STIFFNESS

$(\frac{r}{R})$	(RUNNING MASS/ m_o)
0 - .68	1.0
.69 - .83	1.29
.84 - .96	1.67
.97 - 1.00	0.80

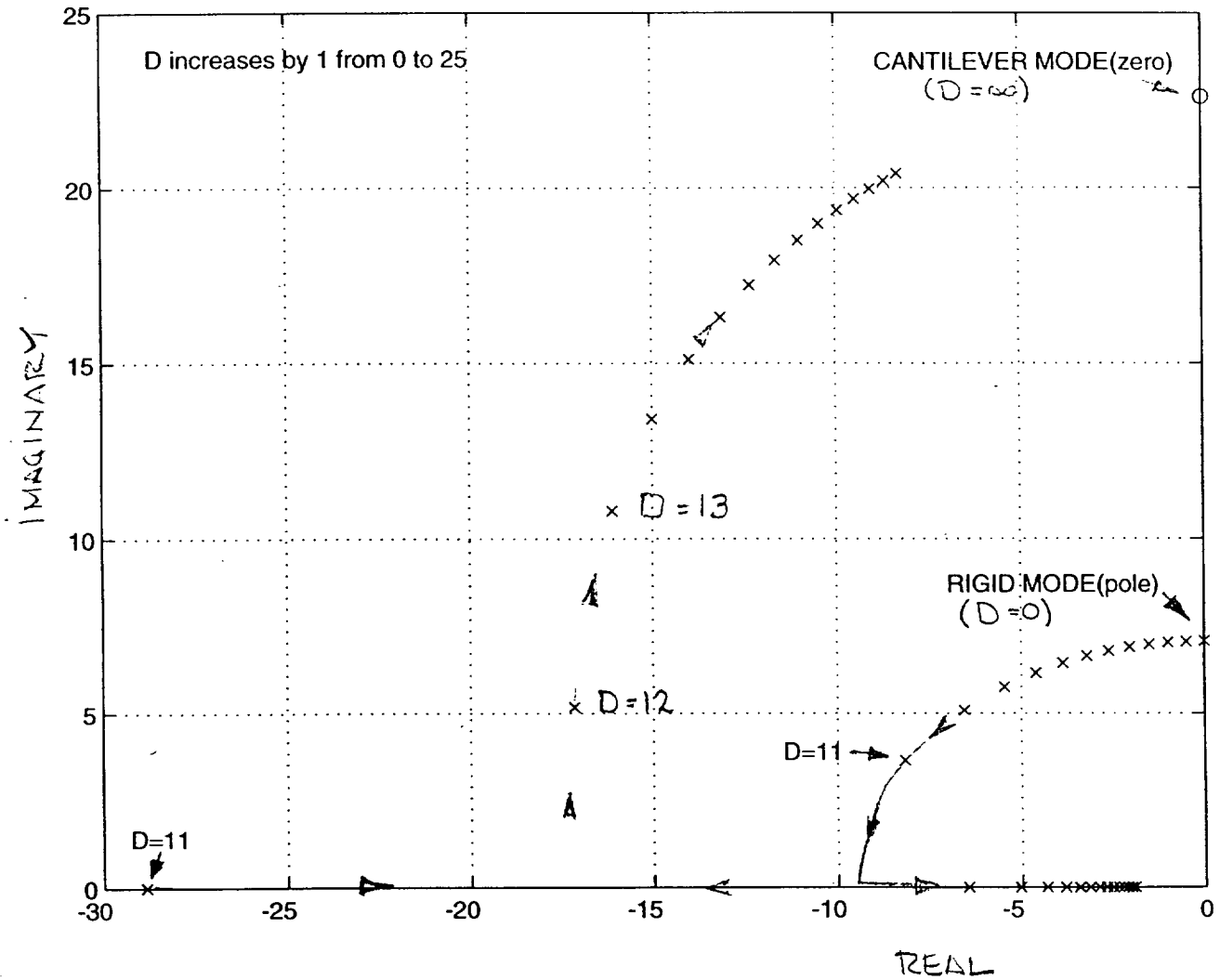
$(\frac{r}{R})$	$(\frac{EI}{EI_o})$
0 - .09	.09
.10 - .14	.43
.15 - .22	.57
.23 - .63	1.00
.64 - .86	.86
.87 - 1.00	.60

$m_o = 0.21 \text{ slugs/ft}$

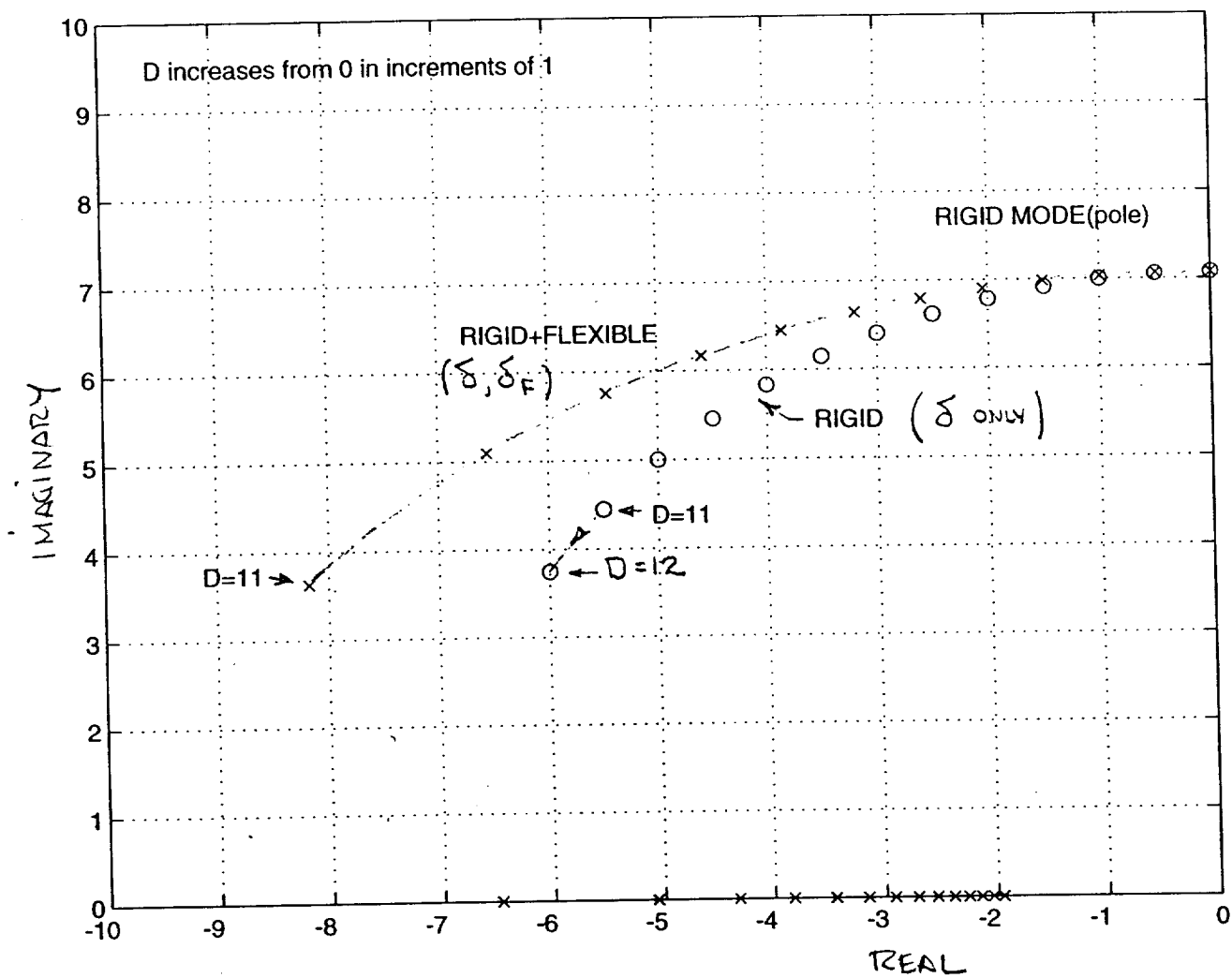
$EI_o = 5.8 \times 10^6 \text{ lb-ft}^2$

TABLE B-1 SPANWISE VARIATION
OF RUNNING MASS AND CHORDWISE STIFFNESS

* HIGH-FREQUENCY PART OF LOCUS
NOT SHOWN (POLES @ $\pm 153.8i$)

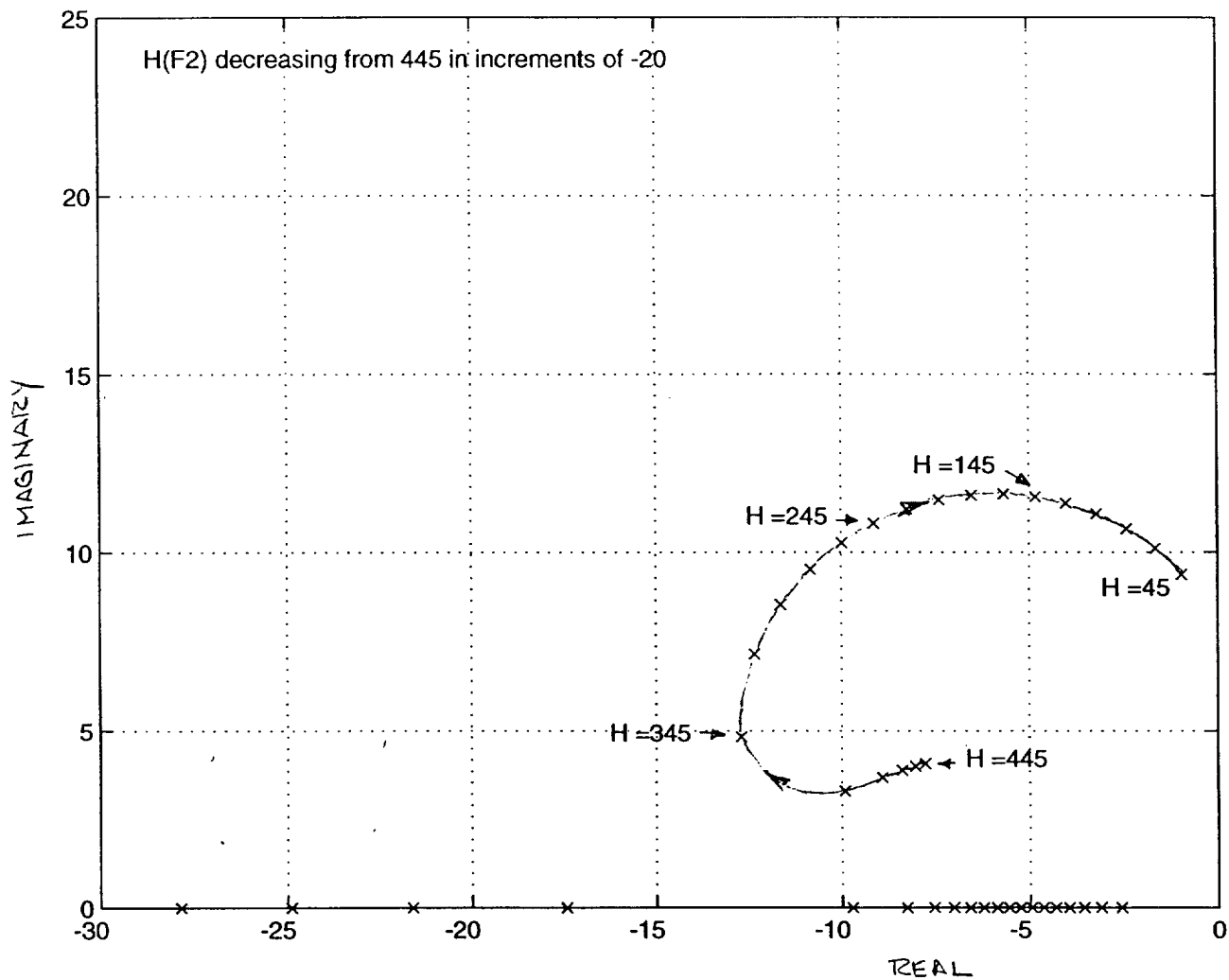


ROOT LOCUS FOR LAG DAMPING INCREASE (D INCREASING)
FIGURE B-1a



ROOT LOCUS FOR LAG DAMPING INCREASE
COMPARISON OF RIGID MODEL WITH RIGID
+ FLEXIBLE MODEL.

FIGURE B-1b



ROOT LOCUS FOR STIFFNESS REDUCTION (H DECREASING)

FIGURE B-2

(B-10)



THIS NOT SHOWN ON
FIGURES B-1, B-2

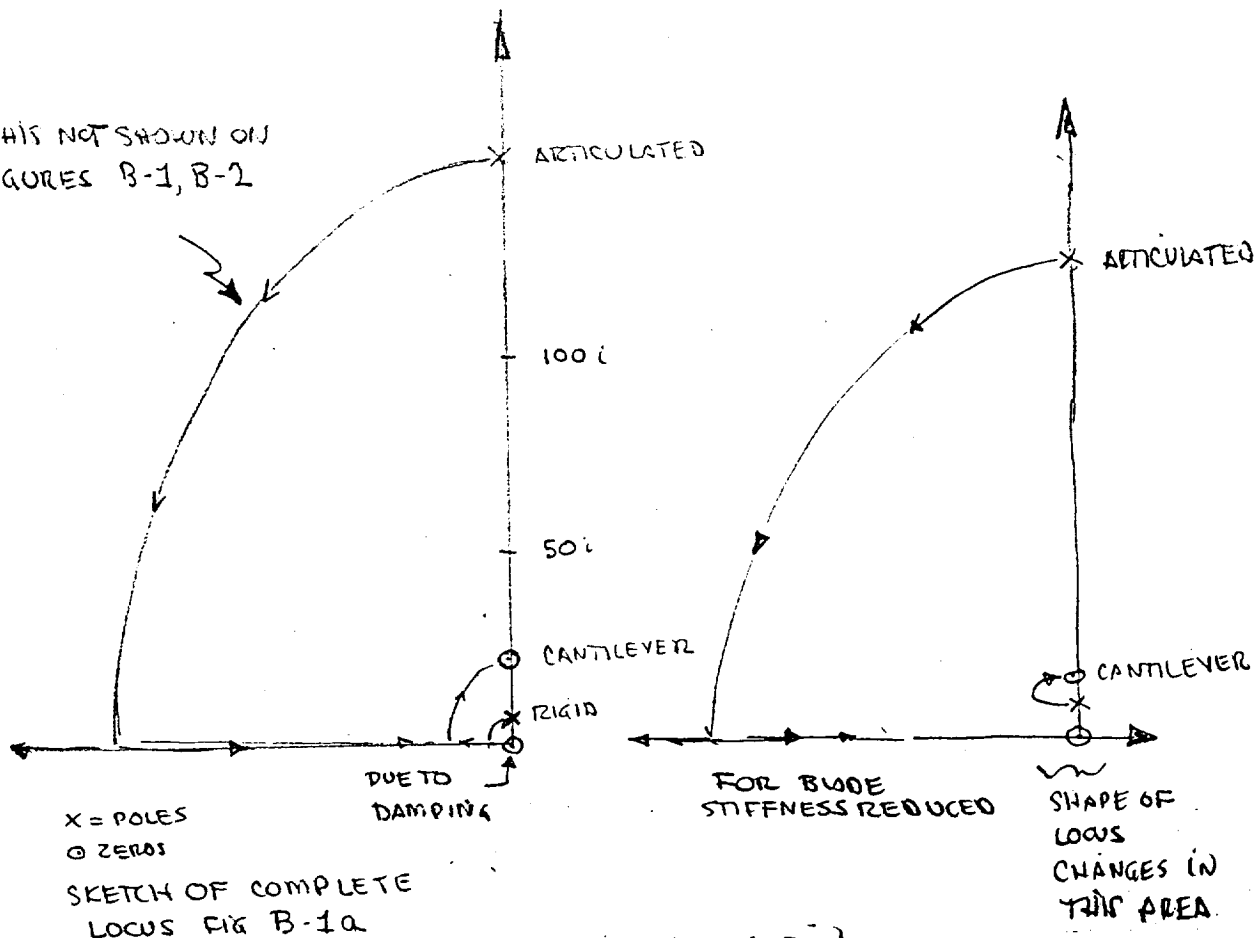


FIGURE B-3

SKETCHES OF B-1a ON A LARGER
SCALE TO SHOW HIGH-FREQUENCY BRANCHES

Appendix C

Publications and Reports

The following is a list of publications prepared under this grant:

Journal Articles

1. Hong,S.W.,and Curtiss,H.C., An Analytic Modeling and System Identification Study of Rotor/Fuselage Dynamics at Hover, J. Mathematical and Computer Modelling, Vol. 19, No. 3/4, February 1994
2. Keller,J., An Investigation of Helicopter Dynamic Coupling Using an Analytical Model, J. American Helicopter Society, Vol.4, No. 4, October 1996 (Lichten Award Paper)
3. Arnold,U.T.P.,Keller,J.D.,Reichert,G.,Curtiss,H.C., The Effect of Inflow Models on the Predicted Response of Helicopters, J. American Helicopter Society, Vol. 43,No.1, January 1998

Conference Papers

1. Curtiss,H.C., Physical Aspects of Rotor Body Coupling in Stability and Control, 46th Annual Forum of the American Helicopter Society, Washington,DC, May 1990
2. McKillip,R.M.,Curtiss,H.C., Approximations for Inclusion of Rotor Lag Dynamics in Helicopter Flight Mechanics Models, 17th European Rotorcraft Forum, Berlin, Germany, September 1991
3. Curtiss,H.C., On the Calculation of the Response of Helicopters to Control Inputs,18th European Rotorcraft Forum, Avignon, France, September 1992
4. Keller,J., Kothmann,B., Hong,S.W., Curtiss,H.C., Modeling the Inplane Motion of Rotor Blades, 5th International Workshop on Dynamics and Aeroelastic Modeling of Rotorcraft Systems, Rensselaer Polytechnic Institute, Troy,NY, October 1993
5. Curtiss,H.C., Some Modeling Issues in Helicopter Flight Dynamics, 20th European Rotorcraft Forum, Amsterdam, Netherlands, October 1994
6. Kothmann,B.D., The Effects of Blade Flexibility on the Control Response of Helicopters, 21st European Rotorcraft Forum, St Petersburg,Russia,September 1995
7. Keller,J., Curtiss,H.C., Modeling the Induced Velocity of a Maneuvering Helicopter, 52nd Annual Forum of the AHS, Washington,DC, June 1996
8. Keller,J., Kothmann,B., Curtiss, H.C., On Aerodynamic Modeling for Rotorcraft Flight Dynamics, 22nd European Rotorcraft Forum, Brighton,UK, September 1996

9. Curtiss, H.C., Aerodynamic Models and the Off-Axis Response, 55th Annual Forum of the AHS, May 1999

Reports/Theses

1. Curtiss, H.C., McKillip, R.M., Coupled Rotor-Body Equations of Motion-Hover Flight, Princeton University Technical Report 1894T, Interim Report, NASA Grant NAG 2-561, June 1990
2. Jensen, P.T., An Analytically Linearized Helicopter Model with Improved Modeling Accuracy, Princeton University Report 1914-T (M.S.E Thesis), Interim Report, NASA Grant NAG 2-561, August 1991
3. Keller, J.D., The Effect of Rotor Motion on the Induced Velocity in Predicting the Response of Rotorcraft, Princeton University Ph. D. Thesis, January 1998
4. MacDonald, B. A., Studies in Helicopter Dynamics Including System Identification Using a Linear Model of a 20,000 lb. Utility Helicopter, Princeton University M.S. E. Thesis, May 1990